CALCULATION OF TEMPERATURE IN THE MOTION OF A PLANE ANNULAR HEAT SOURCE (IN RELATION TO THE DIAMOND DRILLING OF GLASS)

A. N. Reznikov, L. V. Boitsova, and A. V. Temnikov

UDC 536.2:666.1.053.3

The application of the method of heat sources to the problem of the temperature developing in a continuous solid during the motion of an annular plane heat source in the latter is presented. The results are applied to the calculation of the temperature field associated with the drilling of glass with an annular diamond drill.

A number of technical operations involve the problem of calculating the temperature which arises under the influence of a moving annular heat source. This applies in particular to the diamond drilling of glass and other materials with annular drills.

In describing the temperature fields in these and analogous processes two subsidiary problems usually arise: 1) the determination of the temperature due to an annular plane heat source moving in an infinite body; 2) the determination of the temperature at the end of a thin rod (tube) comprising two different materials, one heated at the end, one cooled along the sides. Let us consider the solution of these problems.

A Plane Annular Heat Source in an Infinite Body

Let us suppose that we have an infinite body (Fig. 1) with specified thermophysical properties; at an instant of time $\tau = 0$ a plane annular heat source of intensity q_1 with external and internal radii r_2 and r_1 starts acting in the body, and at the same time starts moving within it at a constant velocity s. The source acts for a time τ. In order to determine the temperature field in the moving system of coordinates xoyz, let us first find the temperature θ_M due to the action of a stationary instantaneous plane annular heat source. For this purpose we use an expression describing the temperature field arising from an instantaneous point heat source:

$$\theta_{pt} = \frac{q'}{\lambda \sqrt{a}\,(4\pi t)^{3/2}} \exp\left[-\frac{R^2}{4at}\right], \tag{1}$$

where

$$R^2 = \rho^2 + \rho_I^2 - 2\rho\rho_I \cos(\mu - \mu_I) + z_0^2 .$$

Let us integrate (1) from 0 to 2π with respect to μ_I and from r_1 to r_2 with respect to ρ_I. We use the equations for an instantaneous circular source given in [1] and obtain

$$\theta_{M(\rho,\mu,z_0,t)} = \frac{q''}{\lambda\sqrt{a}\,(4\pi t)^{3/2}} \int\limits_0^{2\pi}\int\limits_{r_1}^{r_2} \exp\left[-\frac{\rho^2+\rho_I^2-2\rho\rho_I\cos(\mu-\mu_I)+z_0^2}{4at}\right] d\rho_I\rho_I d\mu_I$$

$$= \frac{q''}{8(\pi^3 a^3 t^3)^{1/2} c\gamma} \int\limits_{r_1}^{r_2} \exp\left[-\frac{\rho^2+\rho_I^2+z_0^2}{4at}\right] \left\{\int\limits_0^{2\pi} \exp\left[\frac{\rho\rho_I\cos(\mu-\mu_I)}{2at}\right] d\mu_I\right\} \rho_I d\rho_I$$

V. V. Kuibyshev Polytechnic Institute, Kuibyshev. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 18, No. 1, pp. 154-161, January, 1970. Original article submitted January 3, 1969.

$$= \frac{q''}{4(\pi a^3 t^3)^{1/2} c\gamma} \int_{r_1}^{r_2} \exp\left[-\frac{\rho^2+\rho_I^2+z_0^2}{4at}\right] I_0\left(\frac{\rho\rho_I}{2at}\right)\rho_I d\rho_I$$

$$= \frac{q'' r_2}{2c\gamma\sqrt{\pi a t}} \exp\left[-\frac{z_0^2}{4at}\right] \int_0^\infty \exp[-atm^2] J_0(m\rho) J_1(mr_2)\, dm$$

$$- \frac{q'' r_1}{2c\gamma\sqrt{\pi a t}} \exp\left[-\frac{z_0^2}{4at}\right] \int_0^\infty \exp[-atm^2] J_0(m\rho) J_1(mr_1)\, dm. \quad (2)$$

Here $J_0(m\rho)$, $J_1(mr_1)$, $J_1(mr_2)$ are Bessel functions of the zero and first orders; $I_0(\rho\rho_I/2at)$ is a Bessel function with an imaginary argument. If instead of a stationary coordinate system (x_0, y_0, z_0) we take a moving system (x, y, z) with its origin in the center of the source, moving along the z axis, then $z_0 = z + s(\tau - \tau_i)$; $x_0 = x$ and $y_0 = y$. Let us substitute the value of the coordinate z_0 into Eq. (2) and integrate the expression with respect to τ_i over the limits $\tau_i = 0$ to $\tau_i = \tau$

$$\theta'_M = \int_0^\infty \frac{q_1 r_2}{c\gamma\sqrt{\pi a}} J_0(m\rho) J_1(mr_2) \left\{ \int_0^\tau \frac{\exp\left[-\frac{[z+s(\tau-\tau_i)]^2}{4a(\tau-\tau_i)} - am^2(\tau-\tau_i)\right]}{2\sqrt{\tau-\tau_i}} d\tau_i \right\} dm$$

$$- \int_0^\infty \frac{q_1 r_1}{c\gamma\sqrt{\pi a}} J_0(m\rho) J_1(mr_1) \left\{ \int_0^\tau \frac{\exp\left[-\frac{[z+s(\tau-\tau_i)]^2}{4a(\tau-\tau_i)} - am^2(\tau-\tau_i)\right]}{2\sqrt{\tau-\tau_i}} d\tau_i \right\} dm. \quad (3)$$

We thus obtain an equation for calculating the temperature field due to a plane annular moving heat source at any instant of time τ. For steady heat propagation $(\tau \to \infty)$ we obtain

$$\theta''_M = \int_0^\infty \frac{q_1 r_2}{c\gamma\sqrt{\pi a}} \exp\left[-\frac{sz}{2a}\right] J_0(m\rho) J_1(mr_2) \left\{ \int_0^\infty \exp\left[-\frac{z^2}{4au^2} - \left(\frac{s^2}{4a} + am^2\right)u^2\right] du \right\} dm$$

$$- \int_0^\infty \frac{q_1 r_1}{c\gamma\sqrt{\pi a}} \exp\left[-\frac{sz}{2a}\right] J_0(m\rho) J_1(mr_1) \left\{ \int_0^\infty \exp\left[-\frac{z^2}{4au^2} - \left(\frac{s^2}{4a} + am^2\right)u^2\right] du \right\} dm.$$

Transforming the integral in the curly brackets [2], we finally obtain an expression for the temperature:

$$\theta''_M = \frac{q_1 r_2}{2\lambda} \exp\left[-\frac{sz}{2a}\right] \int_0^\infty \exp\left[-|z| \cdot \sqrt{\frac{s^2}{4a^2} + m^2}\right]$$

$$\times J_0(m\rho) J_1(mr_2) \frac{dm}{\sqrt{\frac{s^2}{4a^2} + m^2}} - \frac{q_1 r_1}{2\lambda} \exp\left[-\frac{sz}{2a}\right]$$

$$\times \int_0^\infty \exp\left[-|z| \cdot \sqrt{\frac{s^2}{4a^2} + m^2}\right] J_0(m\rho) J_1(mr_1) \frac{dm}{\sqrt{\frac{s^2}{4a^2} + m^2}}. \quad (4)$$

In the particular case with $z = 0$ we have

$$\theta = q_1 F_1, \quad (5)$$

where

$$F_1 = \frac{r_2}{2\lambda} \int_0^\infty J_0(m\rho) J_1(mr_2) \frac{dm}{\sqrt{\frac{s^2}{4a^2} + m^2}} - \frac{r_1}{2\lambda} \int_0^\infty J_0(m\rho) J_1(mr_1) \frac{dm}{\sqrt{\frac{s^2}{4a^2} + m^2}}.$$

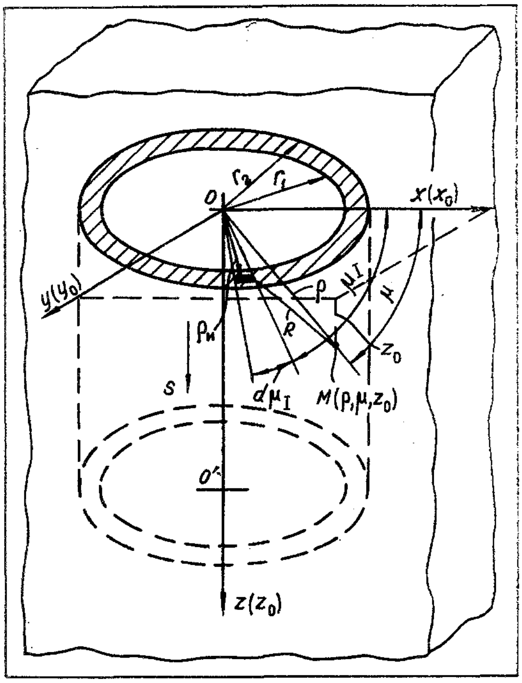

Fig. 1. Diagram to illustrate the calculation of the temperature field due to a plane annular moving heat source.

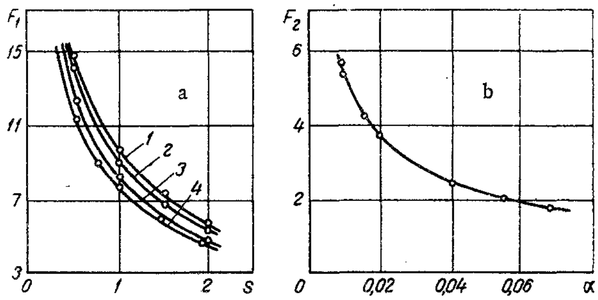

Fig. 2. Set of curves for determining the values of the following functions: a) F_1 (curves 1, 2, 3, 4 refer to source diameters 25, 20, 17, 12 mm); b) F_2. Units are as follows: s in mm/sec, F_1, F_2, and $1/\alpha$ in cal/cm^2·sec·°C.

Curves of the function F_1 are presented in Fig. 2a in relation to the velocity of forward motion of the source. The function F_1 was calculated using the following values in the equation: $r_2 = 6; 8.5; 10; 12.5$ mm; $r_1 = 4.8; 7.3; 8.8; 11.2$ mm, where $\rho = r_2$; $s = 0.3$-2 mm/sec; $\lambda = 0.0023$ cal/cm·sec·°C; $a = 0.0046$ cm^2/sec.

Plane Source at the End of a Composite Thin Cooled Rod

Let us suppose that a rod of length l consisting of two parts l_1 and l_2 with different thermophysical properties lies along the x axis (Fig. 3) and has a constant cross-sectional area f_c and a perimeter H. A heat source of intensity q_2 acts in the plane $x = 0$. The sides give out heat to the surrounding medium. Let us write down the equations of heat conduction and the boundary conditions for this system:

$$\frac{d^2\theta_1}{dx^2} - \mu_1^2\theta_1 = 0 \quad 0 < x < l_1, \tag{6}$$

$$\frac{d^2\theta_2}{dx^2} - \mu_2^2\theta_2 = 0 \quad l_1 < x < l, \tag{7}$$

$$\theta_1 = \theta_c \quad \text{for } x = 0, \tag{8}$$

$$\theta_1 = \theta_2 \quad \text{for } x = l_1, \tag{9}$$

$$\lambda_1\left(\frac{d\theta_1}{dx}\right)_{l_1} = \lambda_2\left(\frac{d\theta_2}{dx}\right)_{l_1} \quad \text{for } x = l_1, \tag{10}$$

$$\left(\frac{d\theta_2}{dx}\right)_l = 0 \quad \text{for } x = l. \tag{11}$$

In Eqs. (6) and (7): $\mu_1^2 = (\alpha H/\lambda_1 f_c$; $\mu_2^2 = (\alpha H/\lambda_2 f_c)$, where α is the heat-transfer coefficient between the rod and the surrounding liquid. The foregoing expressions may also be applied to a tube with walls so thin that the change in temperature across the wall thickness may be neglected. In a tube with thin walls cooled from within and without, the propagation of heat may to a fair approximation be regarded as linear, and the system may be likened to the continuous thin rod illustrated in Fig. 3. For a tube

$$f_c = \pi(r_2^2 - r_1^2); \quad H = 2\pi(r_2 + r_1), \quad \mu_1 = \sqrt{\frac{2\alpha}{\lambda_1 b}}; \quad \mu_2 = \sqrt{\frac{2\alpha}{\lambda_2 b}},$$

where

$$b = r_2 - r_1.$$

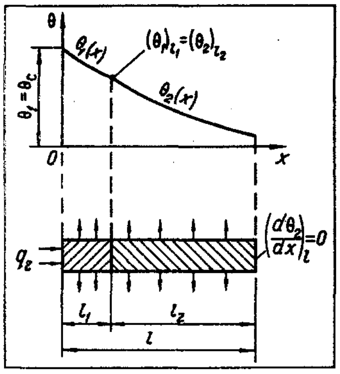

Fig. 3

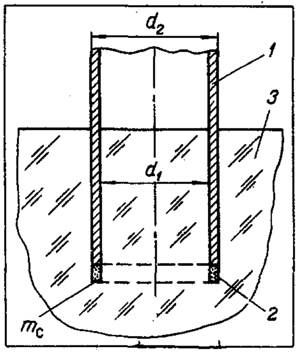

Fig. 4

Fig. 3. Diagram to illustrate the calculation of temperature at the end of a composite thin cooled rod due to a plane heat source. θ in °C.

Fig. 4. Diamond drilling of glass (schematic): 1) body of diamond drill; 2) diamond-carrying layer; 3) glass subject to machining; m_c) working end of drill.

The desired solutions of Eqs. (6) and (7) with the specified boundary conditions take the form

$$\theta_1 = A_1 e^{\mu_1 x} + B_1 e^{-\mu_1 x}; \tag{12}$$

$$\theta_2 = A_2 e^{\mu_2 x} + B_2 e^{-\mu_2 x}; \tag{13}$$

$$A_1 = \frac{Ce^{-p} + D}{C(e^{-p} + e^{n}) + D(1 + e^{n-p})}\,\theta_c;$$

$$B_1 = \frac{Ce^{n} + De^{n-p}}{C(e^{-p} + e^{n}) + D(1 + e^{n-p})}\,\theta_c;$$

$$A_2 = \frac{2e^{\frac{n}{2}+u-2k}}{C(e^{-p} + e^{n}) + D(1 + e^{n-p})}\,\theta_c;$$

$$B_2 = \frac{2e^{\frac{n}{2}+u}}{C(e^{-p} + e^{n}) + D(1 + e^{n-p})}\,\theta_c.$$

The dimensionless quantities C and D have the following meanings:

$$C = 1 + \sqrt{\frac{\lambda_2}{\lambda_1}}; \quad D = 1 - \sqrt{\frac{\lambda_2}{\lambda_1}}.$$

The power indices p, n, u, and k respectively equal

$$p = -\left[\sqrt{\frac{8\alpha}{\lambda_2 b}}\,(l_1 - l)\right]; \quad n = \sqrt{\frac{8\alpha}{\lambda_1 b}}\,l_1;$$

$$u = \sqrt{\frac{2\alpha}{\lambda_2 b}}\,l_1; \quad k = \sqrt{\frac{2\alpha}{\lambda_2 b}}\,l.$$

In order to determine the temperature at the end of the rod, we use the well-known expression

$$q_2 = -\lambda_1\left(\frac{d\theta_c}{dx}\right)_{x=0}.$$

Differentiating Eq. (12), we obtain

$$q_2 = \lambda_1 \mu_1 (B_1 - A_1)$$

and further

$$q_2 = \sqrt{\frac{2\alpha\lambda_1}{b}} \cdot \frac{C(e^n - e^{-p}) + D(e^{n-p} - 1)}{C(e^{-p} + e^n) + D(e^{n-p} + 1)} \theta_c,$$

whence

$$\theta_c = q_2 F_2, \tag{14}$$

where

$$F_2 = \frac{C(e^{-p} + e^n) + D(e^{n-p} + 1)}{C(e^n - e^{-p}) + D(e^{n-p} - 1)} \frac{1}{\sqrt{\frac{2\alpha\lambda_1}{b}}}.$$

A curve representing the function F_2 is given in Fig. 2b in relation to the heat-transfer coefficient α. The function was calculated for values of $\lambda_1 = 0.35$ cal/cm·sec·°C, $\lambda_2 = 0.11$ cal/cm·sec·°C and the source ring width b = 1.2 mm.

Temperature in the Diamond Drilling of Glass

The foregoing two typical problems enable us to describe the propagation of heat in a number of cases, particularly in diamond drilling. A simplified representation of the diamond-drilling process is illustrated in Fig. 4. The cooling liquid passes into the gaps between the outer and inner surfaces of the drill and drilled part. If we remember that, during the processing, the heat is chiefly evolved at the end of the drill m_c (Fig. 4), we may write down the following equation for the tool on the basis of Eq. (14)

$$\theta_c = q_2 F_2. \tag{15}$$

In view of the low thermal conductivity of the glass, the heat source at the end of the tool may be regarded as a plane annular source moving in an infinite body with respect to the material being drilled. Then for an external point on the contact area of the object we may write the following on the basis of Eq. (5)

$$\theta_I = q_1 F_1. \tag{16}$$

Here

$$q_1 + q_2 = q = \frac{P_z v}{0.427 f_c}.$$

It is considered that the whole work of cutting is converted into heat. We do not know in advance how the flux q is divided into q_1 and q_2, but we may determine this by comparing the equations for the temperature at the object/annular drill contact area, based on the conditions applicable to each of the components individually. Comparing Eqs. (15) and (16) for the temperature of points lying on the diameter d_2 and then solving the equations with the two unknowns q_1 and q_2, we obtain

$$q_2 = \frac{qF_1}{F_1 + F_2}; \quad q_1 = \frac{qF_2}{F_1 + F_2}.$$

Hence

$$\theta_c = \frac{P_z v}{0.427 f_c} \left(\frac{F_1 F_2}{F_1 + F_2} \right). \tag{17}$$

By way of example we may present the following data. In the drilling of a plate of commercial glass with an A16M1F12 diamond drill (100% concentration, in one mode the circumferential velocity v = 2.8 m/sec and the axial force P = 20 kG), $P_z = 4.5$ kG. The area of the working end of the annular drill $f_c = 0.408$ cm^2. Let us determine the temperature θ_c arising in this case. From the curves of Fig. 2 we obtain $F_1 = 5.2$, $F_2 = 4.24$, and then by Eq. (17)

$$\theta_c = \frac{4.5 \cdot 2,8}{0.408 \cdot 0,427} \cdot \frac{5.2 \cdot 4.24}{5.2 + 4.24} \approx 170\,°\mathrm{C}.$$

Equation (17) may also be used in the solution of other analogous technical problems.

NOTATION

ρ_I, μ_I	are the polar coordinates of an instantaneous point heat source placed on a circle of radius ρ_I;
p, μ, z	are the coordinates of the point under consideration M;
q'	is the intensity of the instantaneous point heat source;
q"	is the intensity of a plane annular instantaneous heat source;
λ, a	are the thermal conductivity and thermal diffusivity of the material (glass);
$c\gamma$	is the bulk specific heat;
t	is the time from the onset of the instantaneous heat pulse;
$m, u = \sqrt{\tau - \tau_i}$	are the integration variables;
θ_1, θ_2	are the current values of the temperature in different parts of the rod l_1 and l_2 in length;
$b = r_2 - r_1$	is the width of the source annulus;
λ_1, λ_2	are the thermal conductivities of the parts of the rod (tube) l_1 and l_2 respectively;
α	is the heat-transfer coefficient between the rod and the cooling liquid;
P_z	is the circumferential (peripheral) force arising in the diamond drilling of glass;
v	is the rate of rotation of the drill;
s	is the velocity of the source.

LITERATURE CITED

1. H. Carslaw and D. Jäger, Heat Conduction in Solids [Russian translation], Nauka (1964).
2. I. M. Ryzhik and I. S. Gradshtein, Tables of Integrals, Sums, Series, and Products [in Russian], GITTL (1951).
3. A. N. Reznikov, Heat Transfer in Cutting and the Cooling of Tools [in Russian], Mashgiz (1963).